Linear Solvers for Adjoint Problems

Final Public Event
November 25th 2021

Coordinator:
Michaël Méheut (ONERA)

Presenter:
Haysam Telib (Optimad)

MADELEINE project,
Grant Agreement No 769025

Outline

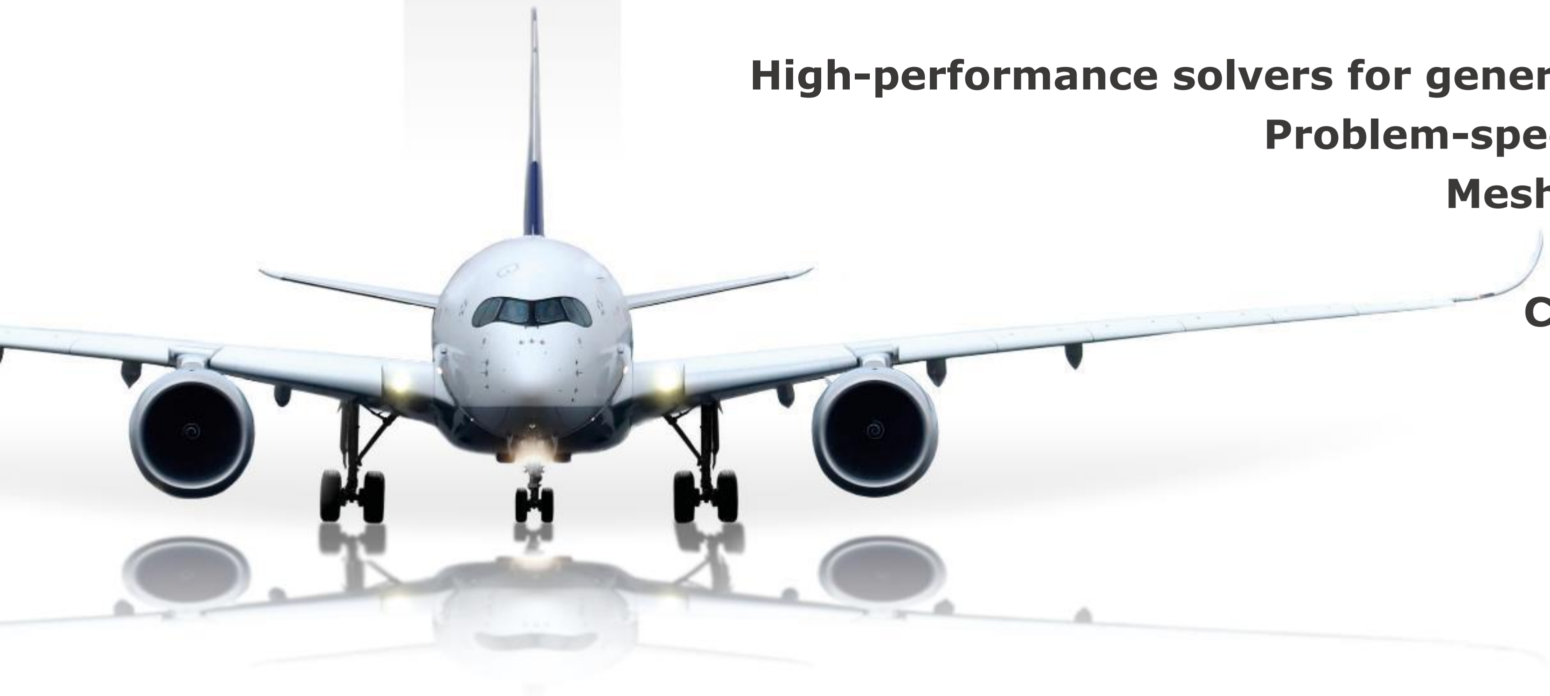
Introduction

High-performance solvers for general adjoints

Problem-specific solver

Mesh morphing

Conclusions



The role of linear solvers

Motivations

Linear solvers are at the heart of adjoint based optimization

- used to solve the discrete adjoint equations
- used to propagate mesh deformation from boundary to entire computational domain

From single to multi discipline (Madeleine)

- used to solve the system of systems of coupled adjoints (disciplinary adjoints + coupling)

Challenges

- **cost -> 10^8 degrees of freedom;**
  - algorithmic complexity
  - memory requirements
  - scaling and distributed computation especially on next-gen HW

vs

- **robustness**
  - convergence in complex situations, problem dependency
  - usability, especially in multi-disciplinary setting
  - accuracy of resulting gradients

Maintainability

Objectives

Efficient MDO process

- accelerate the **execution time** of the MDO process by a **factor of 5**
  by exploiting HPC capabilities, optimize code, develop alternative algorithms

Maintainability

- build upon and integrate into (widely-used) existing framework
- to factorize efforts, especially towards next-gen HW and energy efficiency

Outline

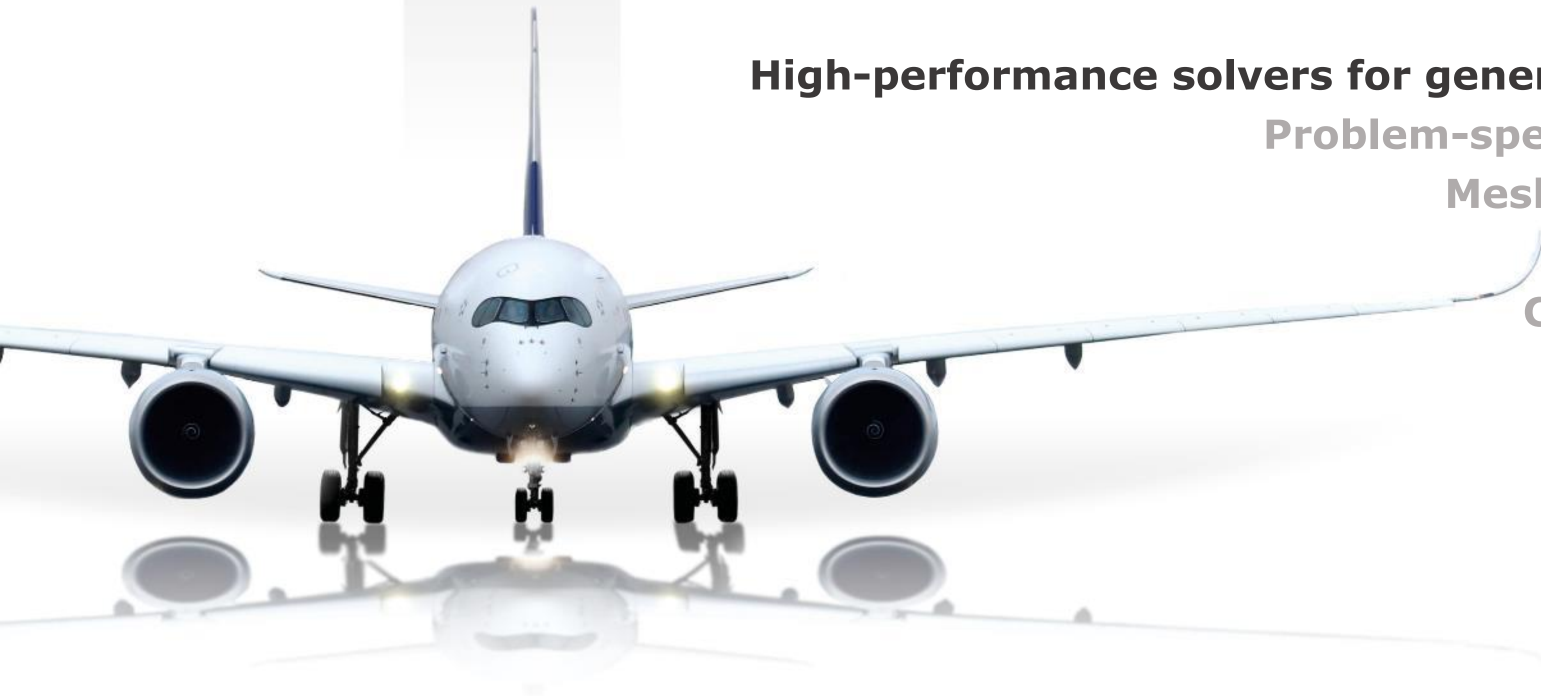
Introduction

High-performance solvers for general adjoints

Problem-specific solver

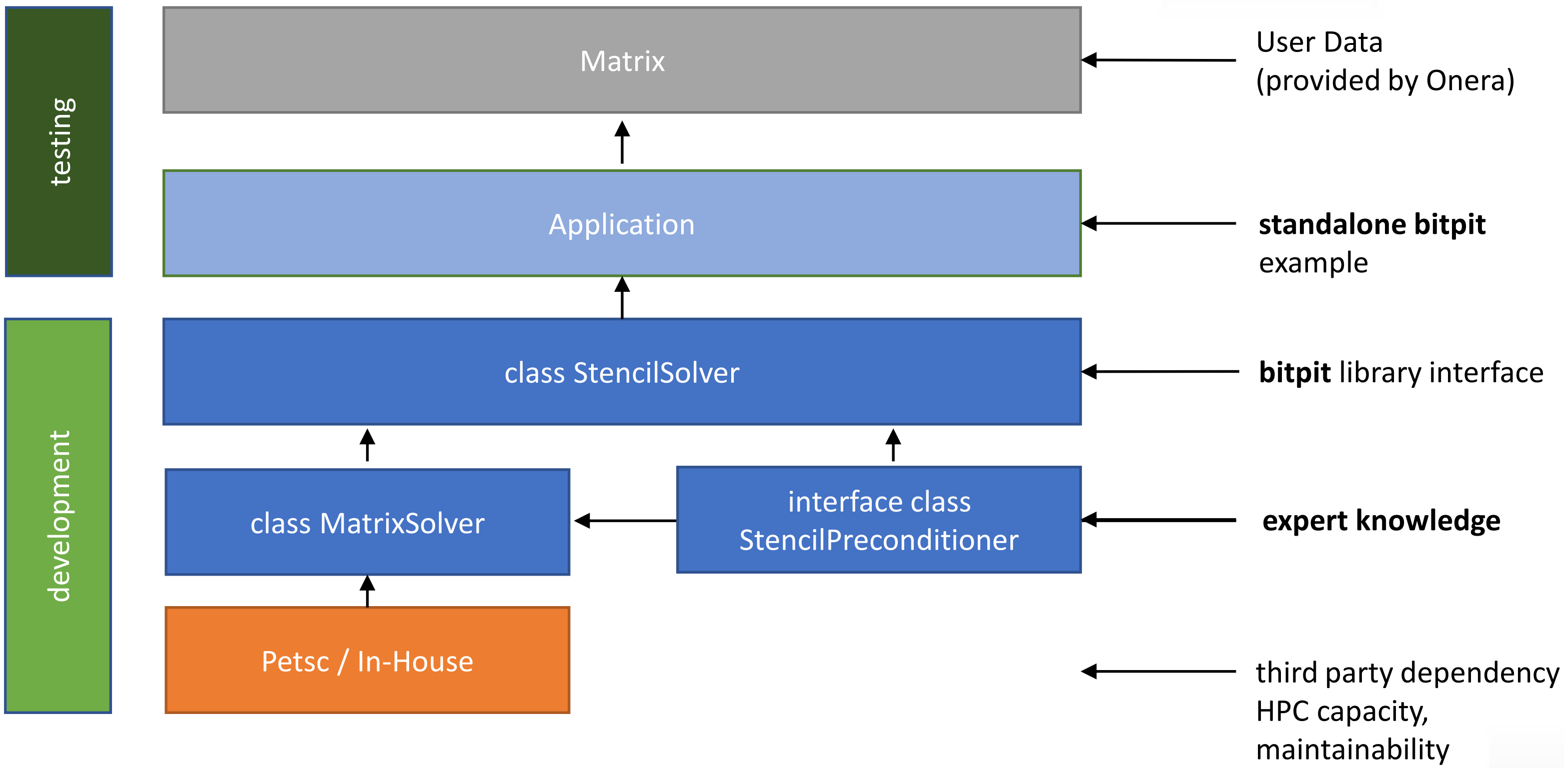
Mesh morphing

Conclusions



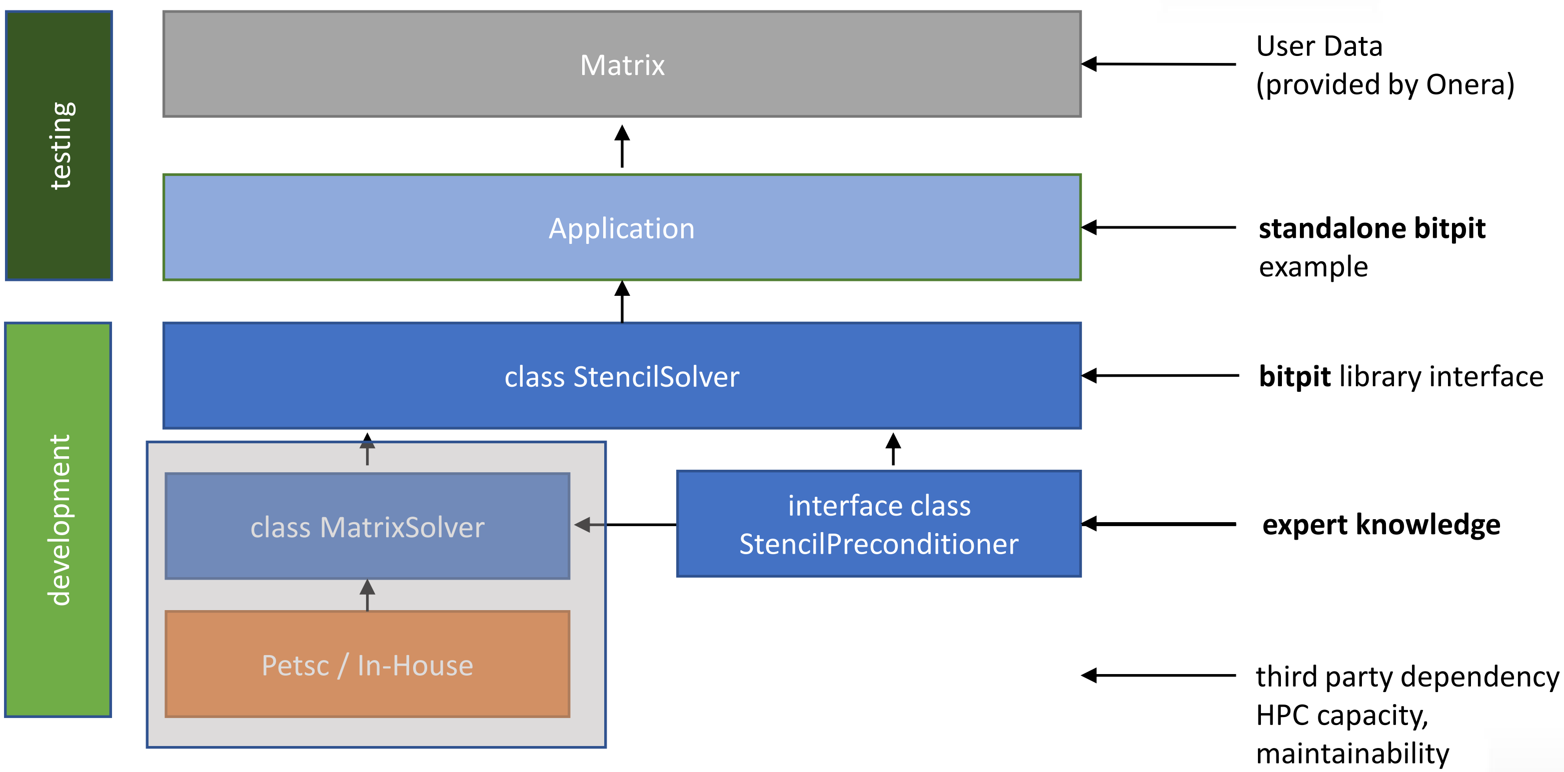
Framework

- Different layers for iterative solvers, preconditioners and testing



1 Optimized iterative solvers

- Exploitation of Flexible GMRES and Deflation techniques



MADELEINE ONERA THE FRENCH AEROSPACE LAB nlr OPTIMAD

Test case

2D compressible RANS equations

Transonic turbulent flow over OAT15A profile

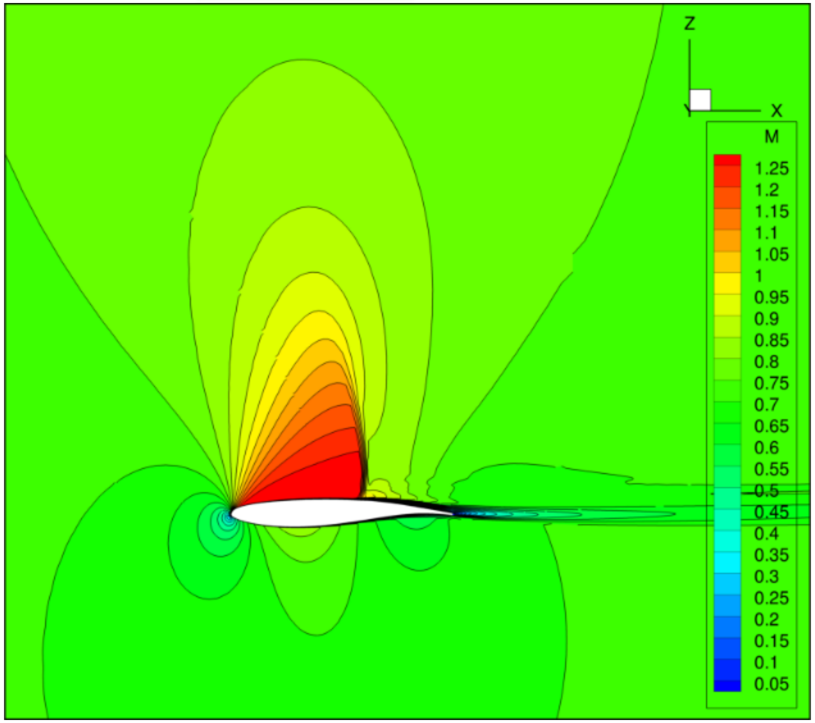
- $M=0.73$, $\alpha=2.5°$, $Re=3 \times 10^6$

Space discretization

- Mesh is composed of 64416 hexahedra
- 4th order DG scheme

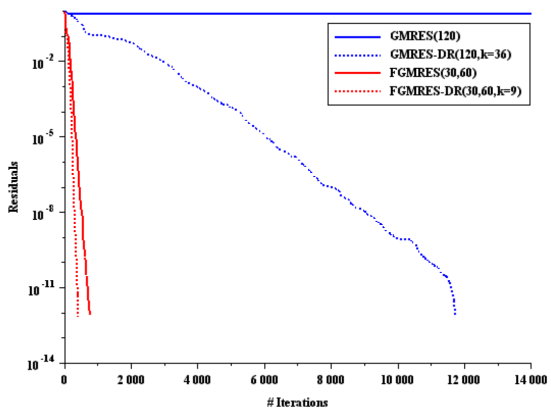
Aerodynamic shape optimization problem

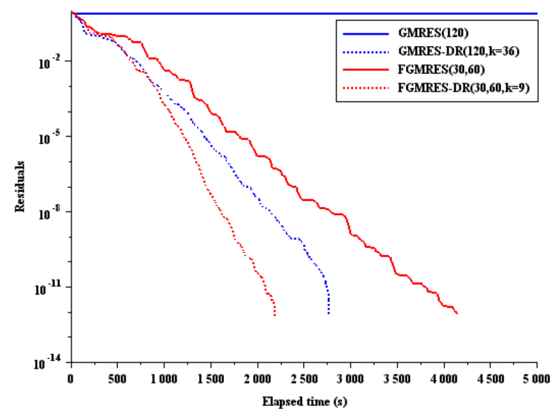
- Minimization of drag coefficient



1 Enhanced iterative solvers, Flexible GMRES +deflation

- 4^{th} order DG scheme with 12 MPI processes
- memory foot-print kept constant



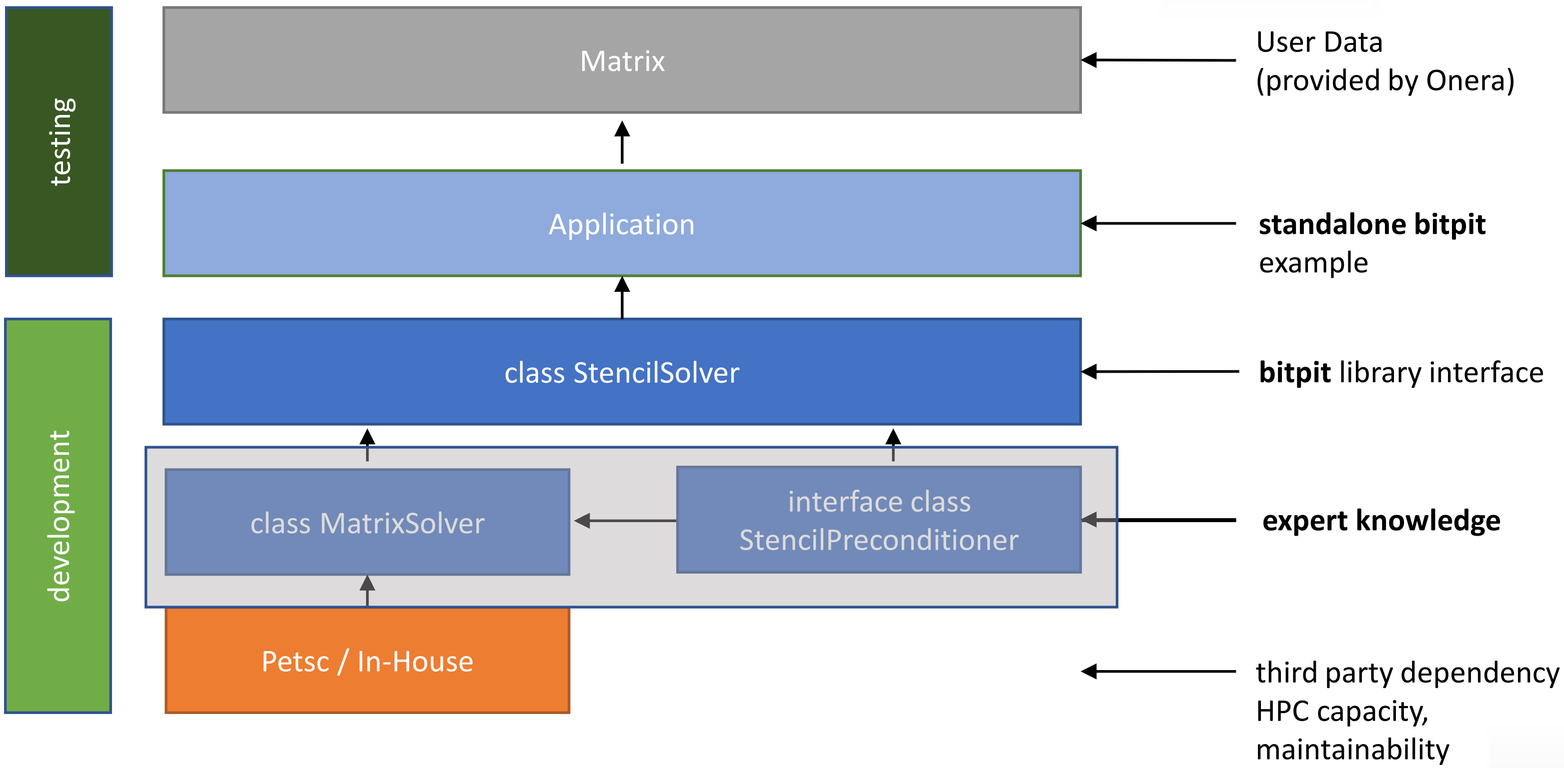


	GMRES(120)		*FGMRES*(30, 60)	
deflation k eig .	—	36	—	9
mixed precision	—	—	yes	yes
iterations	*maxit*	11701	747	394
total execution (s)	×	2767	4136	2186
speedup	×	—	1.00	1.89
speedup	×	1.00	—	1.27

Deflation effect on FGMRES
Best gain over all converging methods

2 Optimized preconditioners

- Problem-aware preconditioning



Test case

2D compressible NS equations

Subsonic laminar flow over NACA0012 profile

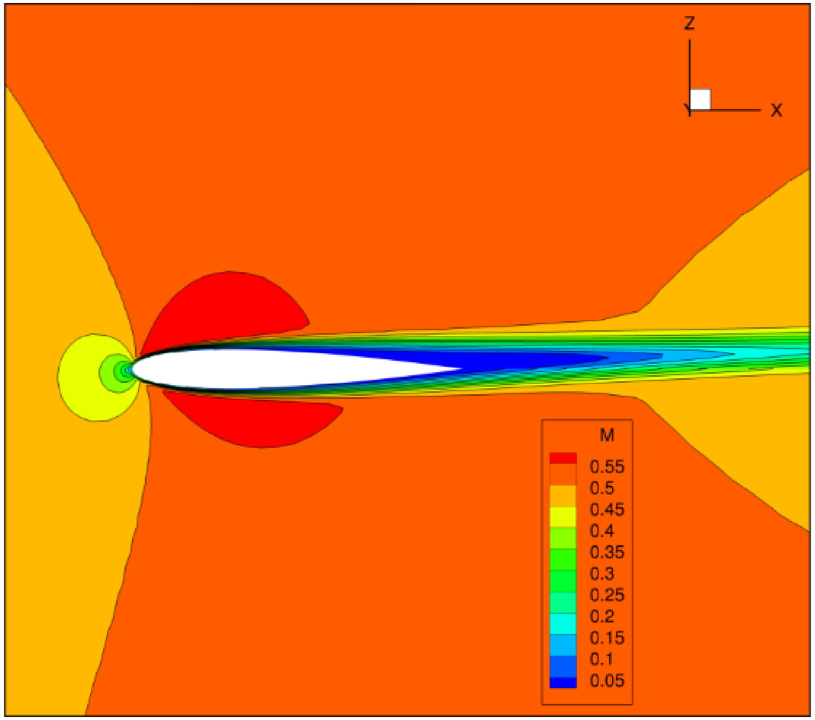
- M=0.5, α=2°, Re=5000

Space discretization

- Mesh is composed of 6400 hexahedra
- 2nd order FV scheme

Aerodynamic shape optimization problem

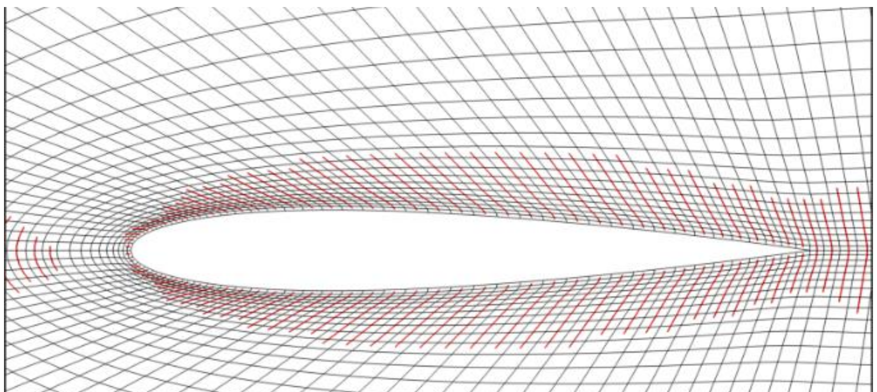
- Minimization of drag coefficient



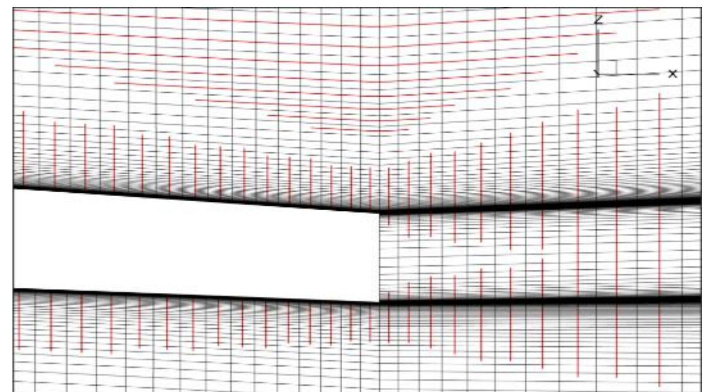
Coupling of degrees-of-freedom: creation of lines

A line construction algorithm to detect strong connections between elements [1]

- Connection's strength is based on the scalar matrix associated to Energy equation
- Each Jacobian entry is associated to a line
- Enlarged inputs for **bitpit** library



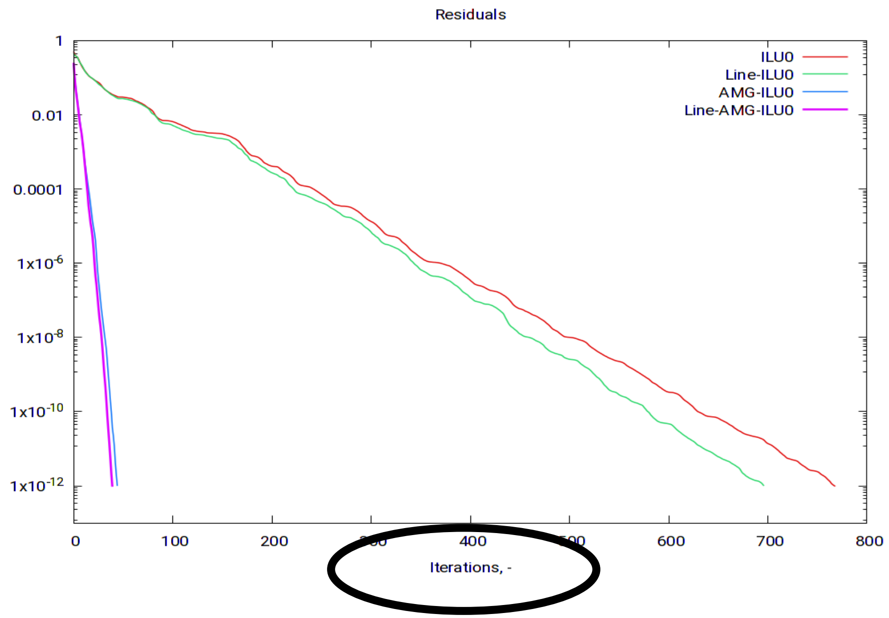
Line construction for NACA0012 laminar flow



Line construction near trailing edge for OAT15A *turbulent flow*

[1] An implicit block ILU smoother for preconditioning of Newton-Krylov solvers with application in high-order stabilized finite-element methods, B. R. Ahrabi, D. J. Mavriplis, Computer Methods in Applied Mechanics and Engineering, 358, 2020.

GMRES + Line-based preconditioner



	Setup time	Solution time	Iterations	Memory	Speedup
ILU0	0.9s	64.6s	769	x1	x 1
Line ILU0	0.9s	53.6s	697	x1	x 1.2
Line AMG	3.9s	26.4s	40	x1.8	x 2.4

MADELEINE ONERA THE FRENCH AEROSPACE LAB nlr OPTIMAD

Outline

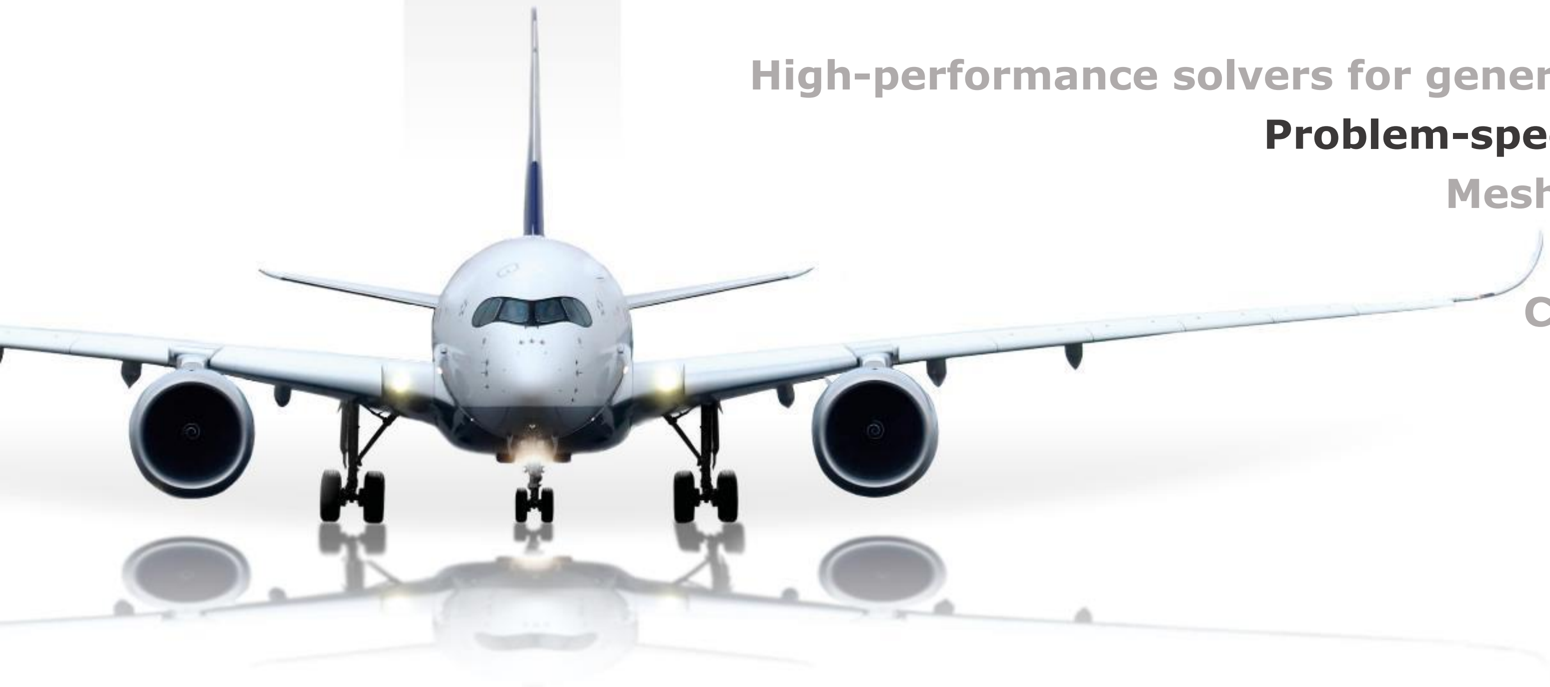
Introduction

High-performance solvers for general adjoints

Problem-specific solver

Mesh morphing

Conclusions

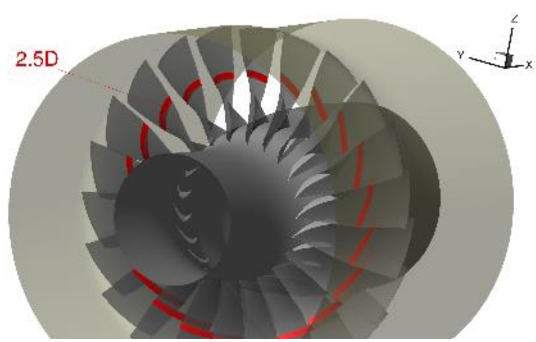


Optimize adjoint for periodic flows with phase lags

For turbomachinery (and propellers)

Zero-phase periodicity
the same phenomena occurs at each blade at each instance
- use periodic boundary conditions on single blade passage

Phase-lagged periodicity ()e.g. distorted inflow)
the same phenomena occurs at each blade but with a phase delay in time
- use full unsteady solver on entire stage



2.5D Rotor 67 with 22 blade passages in a periodically unsteady inflow

Optimize adjoint for periodic flows with phase lags

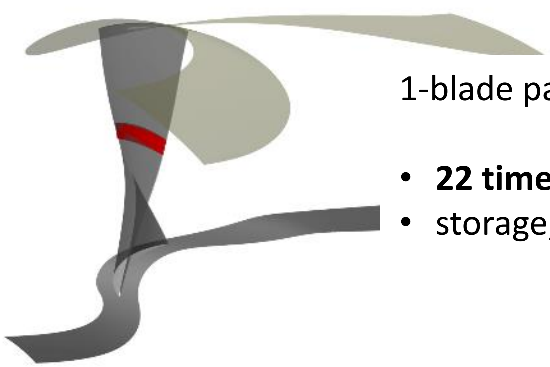
For turbomachinery (and propellers)

Zero-phase periodicity
the same phenomena occurs at each blade at each instance

- use periodic boundary conditions on single blade passage

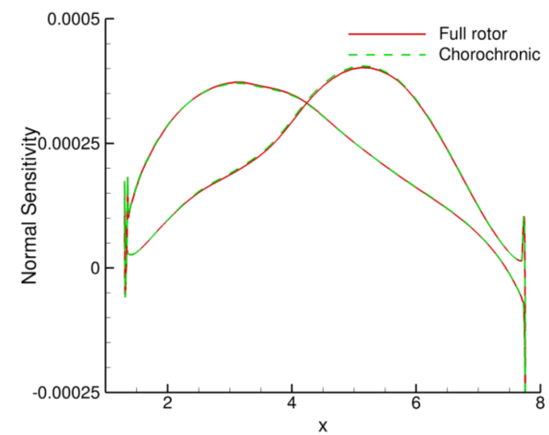
Phase-lagged periodicity ()e.g. distorted inflow)
the same phenomena occurs at each blade but with a phase delay in time

- ad-hoc boundary conditions (**chorochronic bc i.e. space-time boundary condition**)
- exploit efficiently this feature; saving ~number of blade



1-blade passage with chorochronic BC

- **22 times reduction of**
- storage, I/O loads and CPU time



Outline

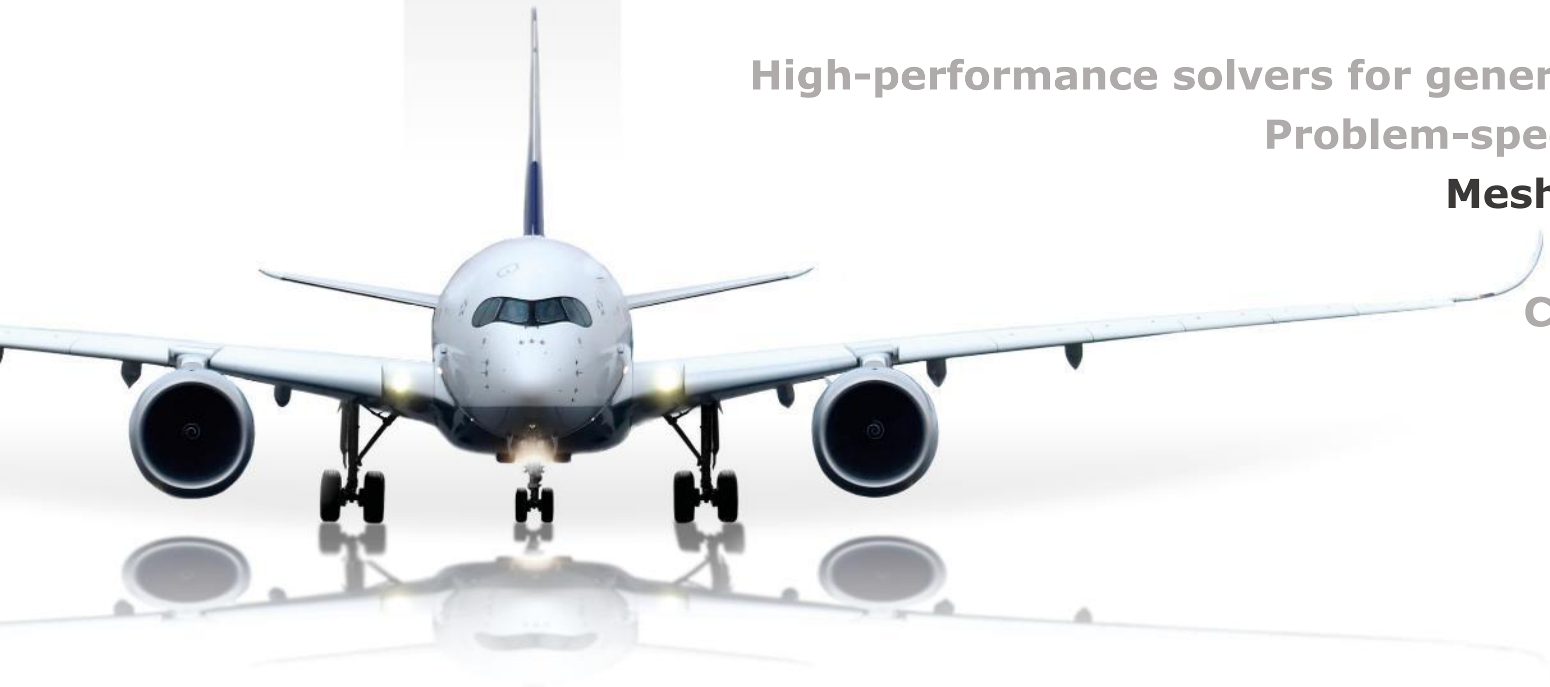
Introduction

High-performance solvers for general adjoints

Problem-specific solver

Mesh morphing

Conclusions



Fast mesh morphing

RBF are graph-free interpolation methods

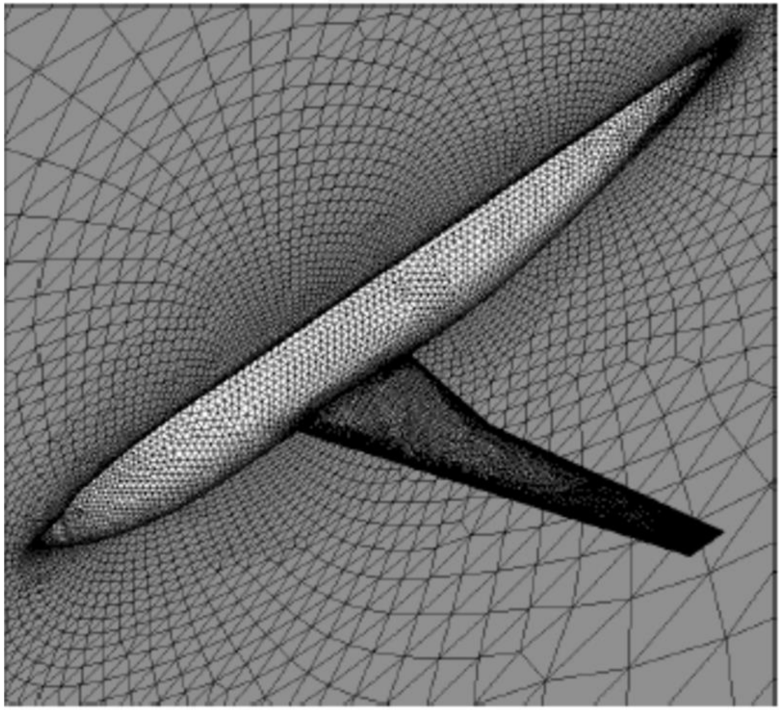
Two steps

1 Finding the interpolant

- N_{surf} x N_{surf} solution of a dense linear system
- Industrial case N_{surf}=**10^5**

2 Applying the interpolant

- N_{vol} x N_{surf} operations
- Industrial case N_{surf}=**10^5, N_{vol}=$10^{7\text{-}8}$**
- complexity ~$N_{vol}^{5/3}$
  x10 mesh-> x50 cost
  x100 mesh -> x2500 cost



Fast mesh morphing

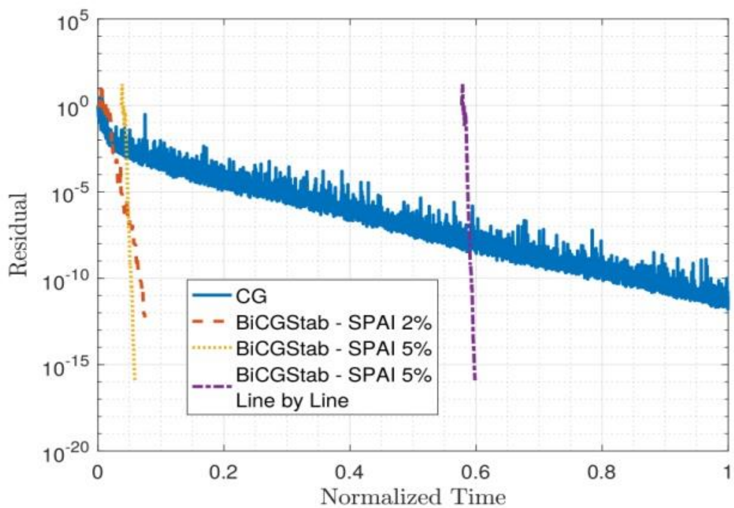
Sparse matrix preconditioner and fast multipole method

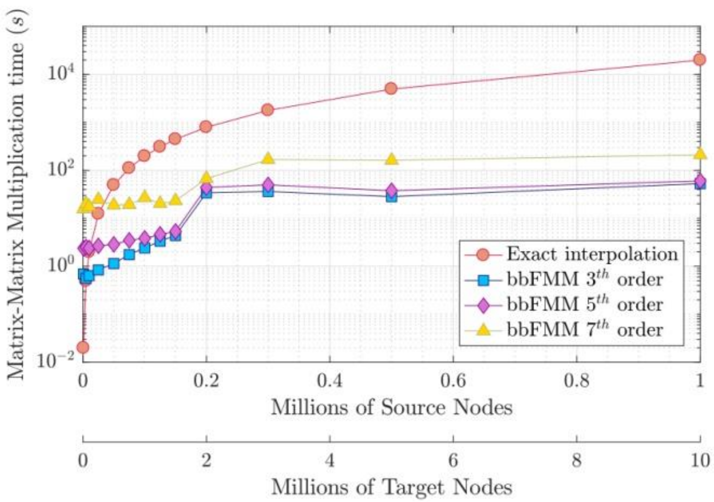
- decompose interpolation
  - global sparse contribution
  - local dense correction

- use optimized algorithms for each stage

overall speed-up >10

recover linear complexity





Outline

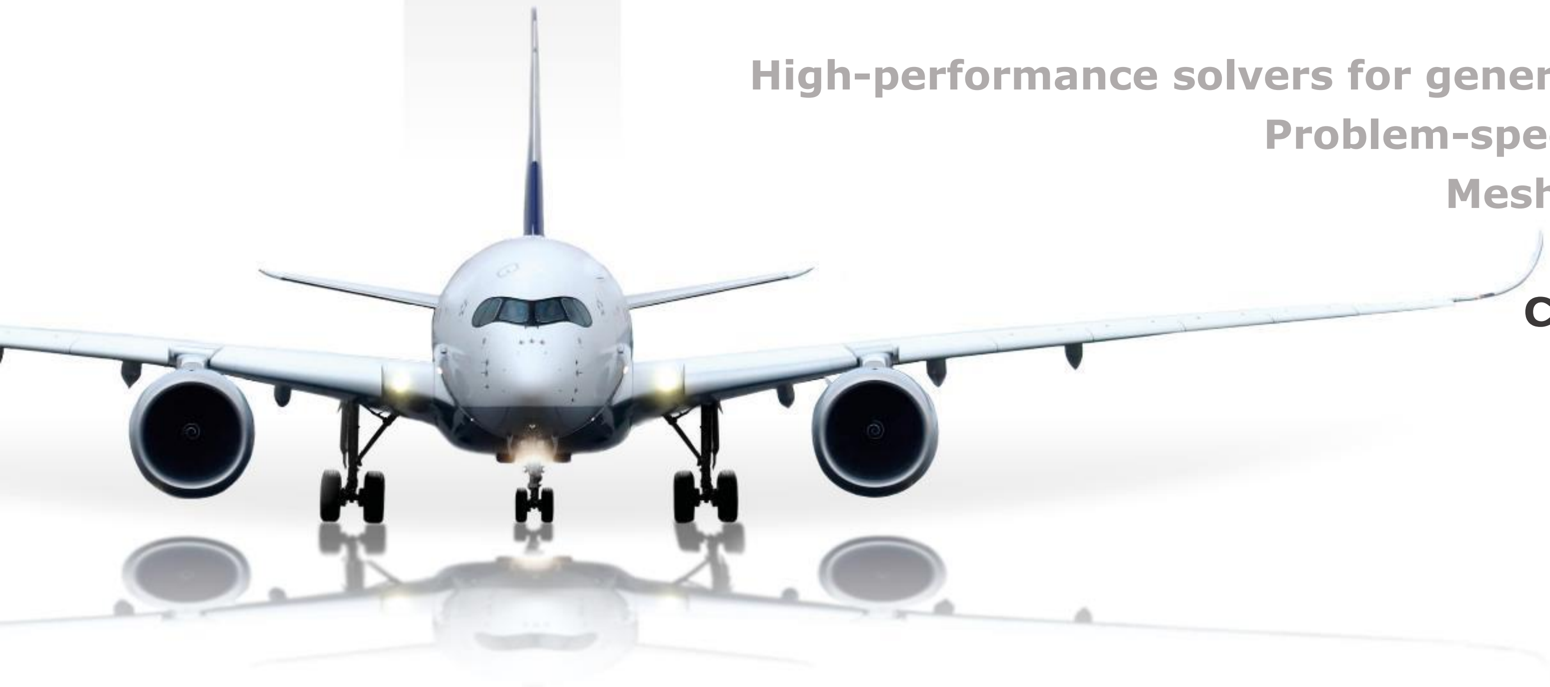
Introduction

High-performance solvers for general adjoints

Problem-specific solver

Mesh morphing

Conclusions



Main achievements & lesson learnt

- speed-up for general linear solvers **around 4** (from iterative methods + preconditioners)
- speed-up for unsteady periodic flows with phase-lag **~number of blades**
- speed-up for RBF based mesh morphing **>10**

- **optimization (domain experts/application specific) & maintainability (compliance with next-gen HW) remains an issue**

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement N° 769025.

www.madeleine-project.eu

www.linkedin.com/company/madeleine-project/